

## Integrated Optical Accumulators with Application in Sigma Delta Modulation

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### Introduction

Optical sigma delta modulation represents a convenient method for achieving high resolution analog-to-digital conversion. Optical architectures for sigma delta offer the capability to extend performance of these devices into the HF band and beyond. One of the issues to be addressed in these types of architectures involves the accumulation of the difference between the comparator feedback signal and the input wideband signal at the Mach-Zehnder interferometer output. Optical accumulation can be performed with two directional couplers and feedback and intra-coupler optical fiber lengths. These fiber lattice structures are capable of performing a wide range of frequency-domain and time-domain signal processing routines. The optical accumulation described here takes advantage of the time-domain matrix multiplication capability.

### Fiber Lattice Structures

The general form of the fiber lattice structure[1] used in the sigma delta modulator described later is shown in Figure 1. The inputs are  $X1$  and  $X2$ ; the outputs  $Y1$  and  $Y2$ . The accumulation rate is controlled by the directional couplers,  $a0$  and  $a1$ , and the optical amplifier denoted by the gain block in the feedback loop. The blocks  $a0$  and  $a1$  are voltage controlled directional couplers. For any particular application the coupling ratio can be fixed, thereby avoiding the need to continuously drive the directional couplers. The optical amplifier is driven by a bistate comparator output which in turn determines whether the optical gain used will accumulate up or down. The difference between the two optical gains is slight, but critical. The delay in the feedback is variable and dependent on the length of the optical fiber. A one pulse period was used in our testing.

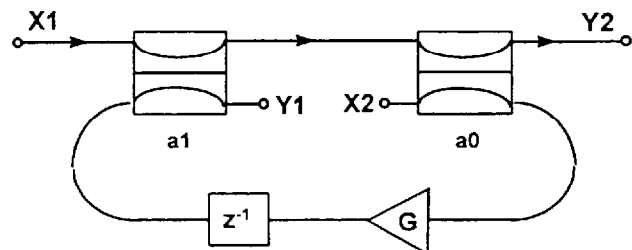


Figure 1: General Fiber Lattice Structure

The transfer functions between either input port and either output port of the general fiber lattice structure are known. The two of interest here are

$$H_{21}(z) = \frac{(1-a_0)(1-a_1)}{1-a_0a_1Gz^{-1}}, \quad H_{12}(z) = \frac{(1-a_0)(1-a_1)Gz^{-1}}{1-a_0a_1Gz^{-1}},$$

where  $H_{mn}(z)$  is the transfer function from input  $X_n$  to output  $Y_m$ . Thus  $H_{21}(z)$  relates the  $X1$  input and  $Y2$  output and  $H_{12}(z)$  relates the  $X2$  input and  $Y1$  output. The  $a_0$  and  $a_1$  represent the percentage of light intensity coupled and therefore are bounded between 0 and 1. The fiber lattice structure configurations for  $H_{12}(z)$  and  $H_{21}(z)$  are shown in Figure 2. The fiber lattice structure  $H_{21}(z)$  is known as a first-order all-pole system and has one zero at the origin and one pole at  $z = a_0a_1G$ . The fiber lattice structure  $H_{12}(z)$  has the delay in the feed-forward path and is known as a first-order pole-zero system. The  $H_{12}(z)$  has the same pole as  $H_{21}(z)$ , but no zeros. The  $H_{12}(z)$  also has greater gain because of the optical amplifier in the feed forward path. Both transfer functions are unstable when  $a_0a_1G > 1$  because the single pole will be located outside of the unit circle. This characteristic is the key to accumulating.

The fiber lattice structures were tested with a step input to determine the relative

response. The optical gain  $G$  was adjusted until the response was a monotonically increasing output. The optical gain necessary to drive either structure to monotonically increase is identical for the same coupling coefficients. For instance, with  $a_0 = 0.5$  and  $a_1 = 0.5$  the optical gain necessary is 6 dB for both  $H_{12}(z)$  and  $H_{21}(z)$ . The difference between the two structures is in the accumulation rate. The  $H_{12}(z)$  accumulates over 10x faster than  $H_{21}(z)$  at these coupling coefficients. Figure 3 shows the amount of optical gain necessary to drive either fiber lattice structure to a monotonically increasing response. The optical gain and accumulation rate decrease as either coupling ratio increases. Figure 4 was generated by switching the feed-back optical amplifier at the same time for each configuration. The steady-state gain characteristics for identical front end gain, feedback gain, and coupling coefficients are shown. Again,  $H_{12}(z)$  accumulates faster than  $H_{21}(z)$  and settles at a greater level because of the feed-forward optical amplifier.

Another series of tests were done to determine the optical gain necessary in order for the  $H_{12}(z)$  fiber lattice structure to settle at the value of the step input. This testing determined the coupling coefficients, front end gain, and feedback gain used in the first-order sigma delta modulator. The result was a toeplitz matrix. This means that once a pair of coupling

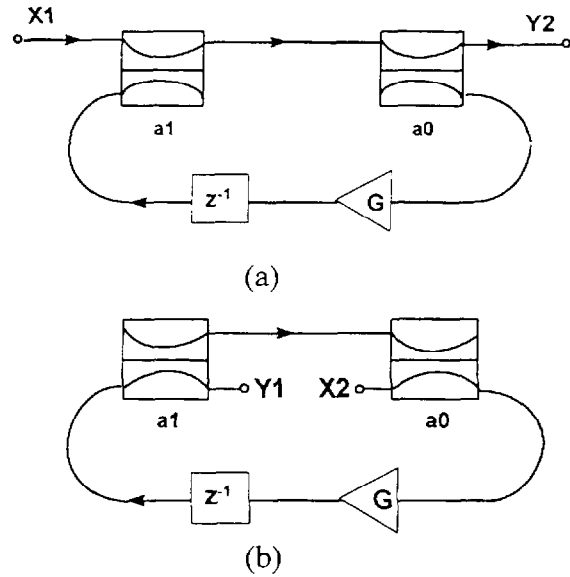


Figure 2: Specific Fiber Lattice Structures  
(a)  $H_{21}(z)$  (b)  $H_{12}(z)$

coefficients are determined, either coupler can take on either value without affecting the result. For example, if the coupling coefficients 0.7 and 0.3 are to be used, either  $a_0$  and  $a_1$  can be set to 0.7 or 0.3 and the results will be identical. These results demonstrated the resiliency of these fiber lattice structures.

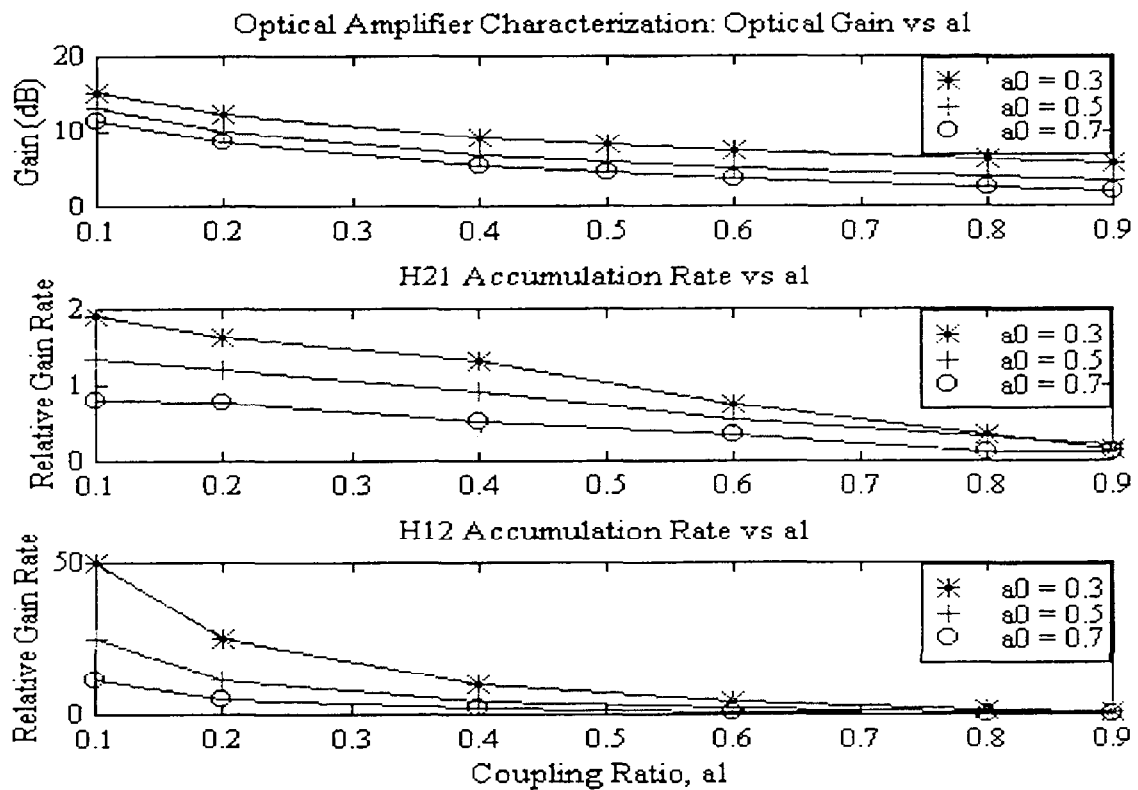


Figure 3: Fiber Lattice Structure Gain Characteristics

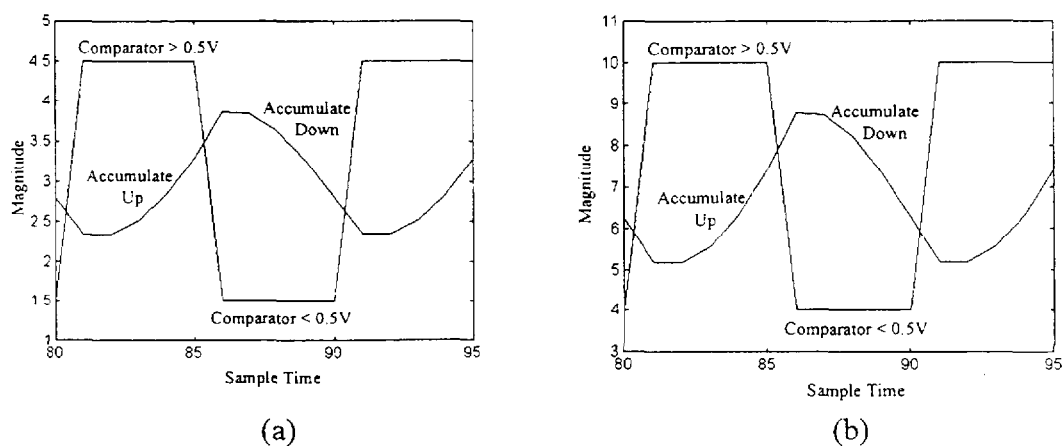


Figure 4: Accumulation Example (a)  $H_{21}(z)$  (b)  $H_{12}(z)$

## First-Order Sigma Delta Modulator

The  $H_{12}(z)$  fiber lattice structure is used in the first-order Sigma Delta modulator. The general configuration is shown in Figure 5. The input to the model is a ramp with evenly spaced numbers between  $-1$  and  $1$ . The Mach-Zehnder interferometers (MZI) are both fed with the same two signals, the input and the comparator feedback. The directional MZI is DC biased with a  $-\pi/2$  voltage while the magnitude MZI is biased with a  $\pi$  voltage. This bias ensures that, when the output of the directional MZI is greater than  $0.5$ , the input to the magnitude MZI is greater than  $0$ . The relationship is shown in Figure 6. The front end gain and the optical gain in the feedback loop of the  $H_{12}(z)$  block are both optical amplifiers with the same gain. The comparator has a threshold of  $0.5$  V. If the signal into the comparator is greater than the threshold, the comparator will set the output to  $0.7$  V. Otherwise, the comparator sets the output to  $-0.7$  V. Either way, the comparator output is fed back to the MZIs and subtracted from the next sampled input signal. At the output of the comparator, the result will be a quantized signal represented by an idling pattern as shown in Figure 7c.

The results of the first order sigma delta modulator were compared to the ideal results known a priori [2]. The ideal model did not account for the feedback optical amplifier; therefore, additional tuning was necessary for the modulator to work under the otherwise same conditions. The conditions for the results shown in Figure 7 were  $a0 = 0.7$ ,  $a1 = 0.3$ , front end gain =  $1.74$ , and the feedback optical amplifier was switched between  $1.74$  and  $1.5$  for the accumulate up and accumulate down condition, respectively.

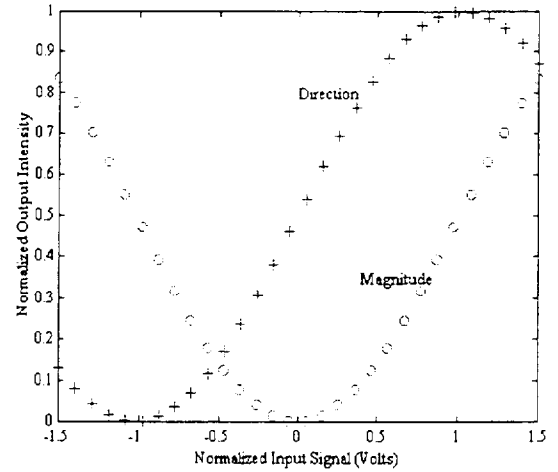


Figure 6: MZI Transfer Functions

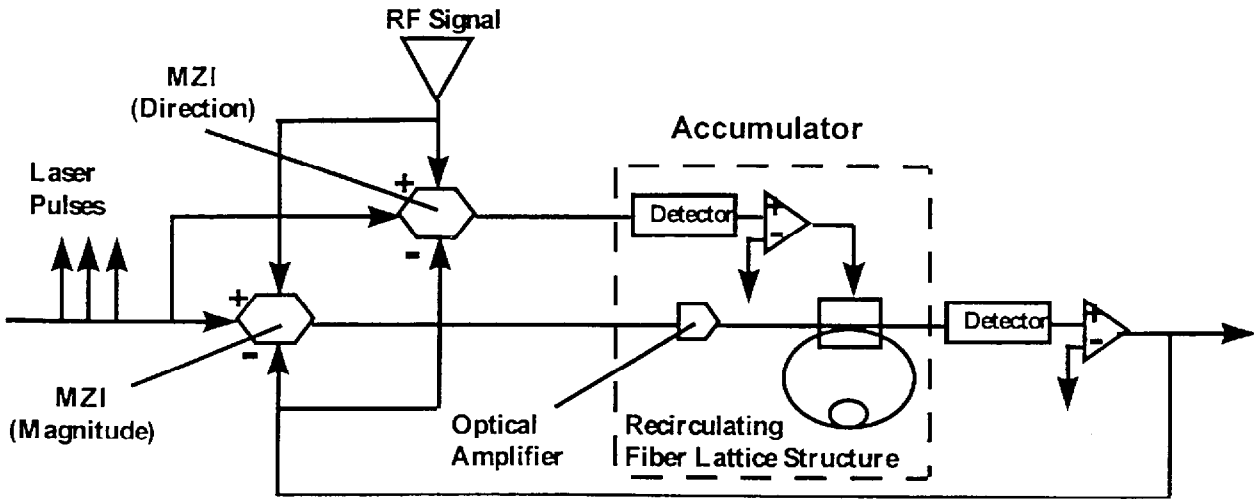


Figure 5: First Order Sigma Delta with Fiber Lattice Structure  $H_{12}(z)$

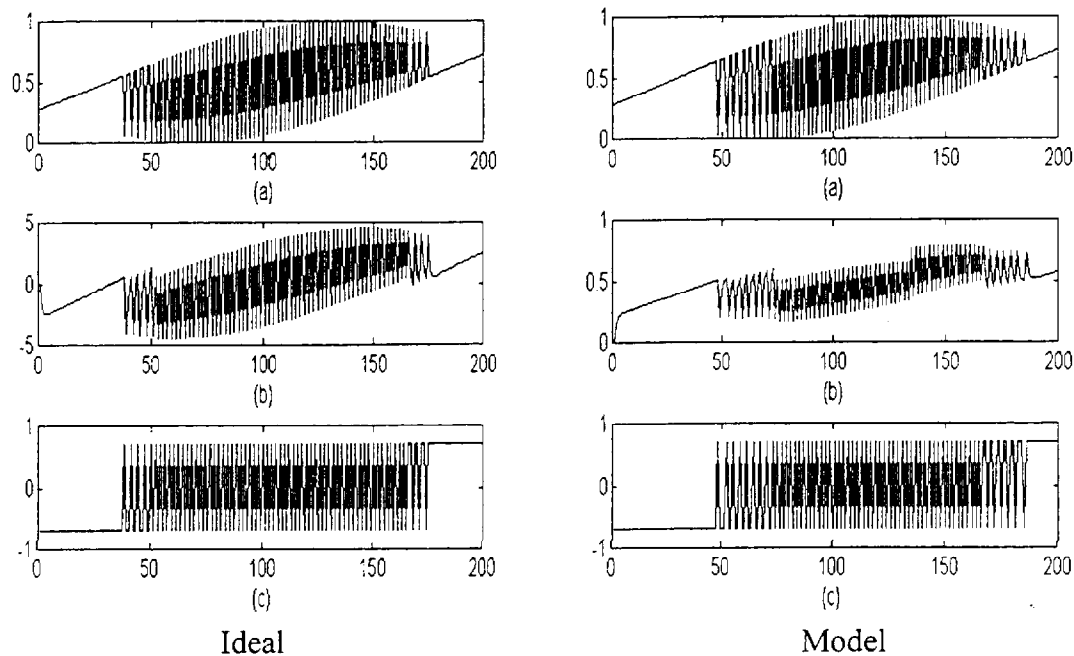


Figure 7: Comparison of Ideal and First Order Model  
(a)  $MZIdir$  out (b)  $H_{12}(z)$  out (c) Discrete Output

The directional MZI output is computed with the overlap parameter,  $\Gamma$ , set equal to 1. At realistic values the result will be flatter, but will still ramp up. The output of the integrator in the ideal case is greater than the model because a front end gain of 50 was used. This has no effect on the  $MZIdir$  and Discrete Output plots in Figure 7 because of the comparator threshold. The idle patterns are similar except the model does not integrate as soon as the ideal case. Increasing the value of the optical gain in the feedback when accumulating down will rectify this slight disparity.

### Conclusion

The two accumulators discussed have been rigorously tested in several ways and results documented. The insertion of the  $H_{12}(z)$  accumulator into the first order sigma delta structure closely follows predicted behavior. Second order sigma delta results should be achievable with insertion of both accumulators into the second order fiber lattice structure and some additional fine tuning. Together, the two should complement each other by increased damping and stability.

### References

1. B. Moslehi, J.W. Goodman, M. Tur, and H.J. Shaw, "Fiber-optic lattice signal processing," *Proc. IEEE*, Vol 72, no. 7, pp. 909-930 (1984).
2. P.E. Pace, S.J. Ying, J.P. Powers, and R.J. Pieper, "Integrated Optical sigma-delta modulators," *Optical Engineering*, 35(7), pp 1828-1836, (1996)